

1. Simulate S samples from the composition ($W^{\parallel(s)}$) using the observed data (Y).

$$W_{.i}^{\parallel(s)} \sim \text{Dir}(Y_{.i} + \alpha)$$

	Sample 1:	Sample 2:	Sample N:
	0.16	0.091	▪ ▪ ▪ 0.12
Taxa 1:	0.14	0.11	▪ ▪ ▪ 0.09
Taxa 2:	0.20	0.15	▪ ▪ ▪ 0.18
▪	▪	▪	▪
▪	▪	▪	▪
▪	▪	▪	▪
Taxa D:	0.05	0.03	▪ ▪ ▪ 0.03

s=1

Original

Updated

2. Apply the CLR transform.

$$\log \hat{W}_{.i}^{(s)} = \log W_{.i}^{\parallel(s)} - \text{mean}(\log W_{.i}^{\parallel(s)})$$

-1.83	-2.40	▪	▪	▪	-2.12
-1.97	-2.21	▪	▪	▪	-2.41
-1.61	-1.89	▪	▪	▪	-1.72
▪	▪	▪	▪	▪	▪
▪	▪	▪	▪	▪	▪
-2.99	-3.51	▪	▪	▪	-3.50

-

-2.25	-2.26
-2.29	32
▪	25
▪	25
-2.30	▪

=

0.43	-0.08	▪	▪	▪	0.13
0.28	0.08	▪	▪	▪	-0.11
0.64	0.39	▪	▪	▪	0.58
▪	▪	▪	▪	▪	▪
▪	▪	▪	▪	▪	▪
-0.75	-1.21	▪	▪	▪	-1.21

2a. Draw samples from the scale model.

$$\log W^{\perp(s)} \sim Q$$

2b. Combine scale samples with composition samples. $\log \hat{W}_{.i}^{(s)} = \log W_{.i}^{\parallel(s)} + \log W_{.i}^{\perp(s)}$

-1.83	-2.40	▪	▪	▪	-2.12
-1.97	-2.21	▪	▪	▪	-2.41
-1.61	-1.89	▪	▪	▪	-1.72
▪	▪	▪	▪	▪	▪
▪	▪	▪	▪	▪	▪
-2.99	-3.51	▪	▪	▪	-3.50

+

-3.25	-4.12
-4.69	58
▪	25
▪	25
-3.58	▪

=

-5.08	-6.90	▪	▪	▪	-6.37
-6.09	-6.90	▪	▪	▪	-5.99
-5.73	-6.58	▪	▪	▪	-5.30
▪	▪	▪	▪	▪	▪
▪	▪	▪	▪	▪	▪
-7.11	-8.20	▪	▪	▪	-7.08

3. For each entity and sample s, compute log fold changes ($\hat{\theta}_d^{(s)}$) and test for an effect.

$$\hat{\theta}_d^{(s)} = \text{mean}_{i \in \text{case}} \log \hat{W}_{di}^{(s)} - \text{mean}_{i \in \text{control}} \log \hat{W}_{di}^{(s)}$$

$$H_0 : \theta_d^{(s)} = 0 \text{ versus } H_A : \theta_d^{(s)} \neq 0$$

4. Aggregate test results across the S samples.